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RE-EVALUATION OF STRENGTH AND STIFFNESS RELATIONSHIPS FOR HIGH-STRENGTH CONCRETE

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ABSTRACT

It has been generally accepted that the tensile strength and elastic modulus of concrete is proportional to the square-root of its compressive strength. This relationship, however, may not be applicable for high-performance concrete. The study presents data on strength and stiffness of concretes containing a laboratory produced metakaolin and commercial silica fume as cement replacement materials, with water-to-cementitious materials ratio of 0.27 to 0.33. Approximately 750 specimens were tested and compressive strength of up to 110 MPa at 90 days were reported. Analysis of the best-fit relationships for tensile-compressive strength and stiffness-compressive strength found that the square-root function recommended by most codes of practice is inadequate when applied to concretes of higher strength, particularly in the case for tensile strength prediction.

Keywords: compressive strength; high-performance concrete; metakaolin; elastic moduli; silica fume; tensile properties

1. INTRODUCTION

Application of high-performance concrete (HPC) has become increasingly popular in concrete construction, particularly for high-rise and marine structures in recent years. Whilst the current research trend on concrete is more focussed on its performance in extreme environments i.e. on durability assessment, one must not forget that strength is still the critical parameter in the design stage of a structure. Ironically, a survey of literature indicates that little has been done to establish a fundamental understanding of the development and interrelationships of various physical properties of HPC.

Concrete is not normally designed to carry load in tension, hence its tensile strength is generally considered as a negligible parameter. However, the knowledge of tensile strength is of substantial importance in concrete structures particularly with regards to crack mitigation. Tensile strength is used to resist shear forces in unreinforced sections; to control cracking in prestressed concrete; and to resist shrinkage and thermal stresses. For

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serviceability limit states, tensile strength is often a more important parameter than compressive strength. It is the limiting factor for safety of many practical applications such as unreinforced concrete structures, mass concrete dams, structures under seismic loadings, highways, airfield pavements and segmental prestressed concrete bridges. In addition, knowledge of elastic properties of concrete is also vital from a serviceability point of view. For reinforced or prestressed concrete, elastic modulus is a basic parameter to evaluate immediate and time-dependant deformations, and prestress losses. An accurate evaluation of stiffness is crucial in assessing the risk of cracking when subjected to thermal gradients and shrinkage.

Table 1: Equations for tensile strength and elastic modulus from various codes of practice, applicable to normal-weight concrete

Code of practice	**Proposed equation**	
Splitting tensile strength		
ACI 318-99 [1]	$f_{sp} = 0.56\sqrt{f_{cyl}}$	$14\text{MPa} < f_{cyl} < 41\text{MPa}$
ACI 363-92 [2]	$f_{sp} = 0.59\sqrt{f_{cyl}}$	$21\text{MPa} < f_{cyl} < 83\text{MPa}$
CEB-FIB MC 90 [3]	$f_{sp} = 1.43 f_{cyl}^{0.67}$	
Flexural tensile strength		
ACI 318-99 [1]	$f_r = 0.62\sqrt{f_{cyl}}$	$14\text{MPa} < f_{cyl} < 41\text{MPa}$
ACI 363-92 [2]	$f_r = 0.94\sqrt{f_{cyl}}$	$21\text{MPa} < f_{cyl} < 83\text{MPa}$
CSA A23.3 [4]	$f_r = 0.6\sqrt{f_{cyl}}$	$20\text{MPa} < f_{cyl} < 80\text{MPa}$
New Zealand Standard [5]	$f_r = 0.8\sqrt{f_{cyl}}$	
Modulus of elasticity		
BS 8110: Part 2: 1985	$E = 9.1 f_{cu}^{0.33}$	$20\text{MPa} < f_{cu} < 60\text{MPa}$
ACI 318-99 [1]	$E = 4.73\sqrt{f_{cyl}}$	$14\text{MPa} < f_{cyl} < 41\text{MPa}$
ACI 363-92 [2]	$E = 3.32\sqrt{f_{cyl}} + 6.9$	$21\text{MPa} < f_{cyl} < 83\text{MPa}$
CSA A23.3 [4]	$E = 3.32\sqrt{f_{cyl}} + 6.9$	$21\text{MPa} < f_{cyl} < 83\text{MPa}$

Despite the importance of having adequate tensile strength and elastic modulus, both properties are seldom, if not ever determined directly on site for compliance purposes. To avoid laborious and time-consuming direct measurements, engineers have often favoured to estimate these values of compressive strength itself based on empirical relationships proposed by various codes of practice; these are summarised in Table 1. Although this empirical approach is usually accurate enough for concretes within the normal strength range, for HPC however, the relationship is not as straightforward. Studies have indicated that the equations put forward were not always applicable to all HPC, suggesting that either the proposed relationships are inadequate, or that each HPC is unique as a result of the various cementitious materials used, water-to-binder ratios and aggregate characteristics, so much so that finding a simple relationship for all types of HPC is almost an impossible task.

The purpose of this paper is to re-examine existing equations for predicting tensile strength and stiffness from compressive strength, by using new data obtained from HPC mixtures incorporating metakaolin and silica fume as pozzolanic microfillers. The obtained values are compared to other published data, and the applicability of existing equations to concretes at higher strength levels is discussed. This study forms part of a larger research programme on the feasibility of calcined Malaysian kaolin as a pozzolan for HPC.

2. EXPERIMENTAL WORK

2.1 Materials

Ordinary Portland cement (ASTM Type 1), laboratory-produced metakaolin and a commercial silica fume was used. The metakaolin was obtained by calcination of refined Malaysian kaolin at 700°C for 7 hours, using a rotary electrical furnace. The specific gravities for cement, metakaolin and silica fume were 3.11, 2.52 and 2.22 respectively. Chemical composition of the cementitious materials is shown in Table 2. A medium graded siliceous sand (BS 882: 1992) and a single-sized 10 mm crushed granite stone was used as fine and coarse aggregates respectively. The specific gravities for fine and coarse aggregate at saturated surface dry condition were 2.65 and 2.57. Particle size distributions for the aggregates are presented in Table 3. A polycarboxylic ether based superplasticizer was used. The dark brown solution has a 20% solids dosage and a specific gravity of 1.05. Mixing water was taken directly from tap supply, at a temperature of 27°C.

Table 2: Chemical composition of cement, metakaolin and silica fume

	SiO_2	Al_2O_3	CaO	Fe_2O_3	MgO	Na_2O	K_2O	P_2O_5	TiO_2	MnO	LOI
Cement	20.99	6.19	65.96	3.86	0.20	0.17	0.60	0.05	0.40	0.06	1.53
MK	57.40	35.26	0.02	0.94	0.18	<0.01	3.17	0.09	0.43	<0.01	2.52
SF	92.06	0.48	0.40	2.11	0.63	0.28	1.24	0.02	<0.01	0.23	2.54

Table 3: Particle size distribution of coarse and fine aggregates

Coarse aggregate (Granite)		Fine aggregate (Siliceous sand)	
Size (mm)	**Percentage passing**	**Size (mm)**	**Percentage passing**
19.0	100	4.75	100
12.7	99.7	2.36	99.9
9.5	97.6	1.18	61.5
6.7	55.5	0.6	49.5
4.75	20.2	0.3	26.5
2.36	0.3	0.15	6.8
		0.075	1.1

2.2 Mixture Proportions

The experimental work covered twenty-one mixtures that were divided into three series: A, B and C with effective water-to-cementitious material (W/CM) ratio of 0.27, 0.30 and 0.33 respectively. Each series consisted of a control and six binary blend mixtures, which were weight replacement of metakaolin or silica fume at 5%, 10% and 15%. All mixtures were designed in accordance to the Sherbrooke Mix Design method [6] for non-air entrained high-performance concrete based on absolute volume. Total cementitious materials content used for all mixtures was 500 kg/m^3 and coarse aggregate content was 1050 kg/m^3. Superplasticizer dosages for Series A, B and C were fixed at 1.8%, 0.8% and 0.5% by weight of cementitious material content respectively. Mixture proportions are summarised in Table 4.

2.3 Specimen preparation

Concrete mixtures were batched using a pan mixer. Three types of specimen were prepared: 100 mm cubes, 100×100×500 mm prisms and 150 Ø x 300 mm cylinders. Specimens were cast in steel moulds and compacted in three uniform layers using vibrating tables equipped with an electronic time controller. The fresh concretes have different slump values because a constant superplasticizer dosage was used for all mixtures in a particular series; however, all were well within a workable range. Vebe time results were used as a guide for the amount of vibration required to ensure proper compaction of the fresh concrete. Specimens were covered with wet burlap for the first 24 hours, after which the moulds were stripped and the specimens were cured in a water tank at 27°C until the day of testing.

Table 4: Mixture proportions

Mixture	Cement (kg/m^3)	MK (kg/m^3)	SF (kg/m^3)	Water (kg/m^3)	W/CM	Granite stone (kg/m^3)	Siliceous sand (kg/m^3)	SP (l/m^3)
Series A (W/CM = 0.27)								
CA	500	-	-	135	0.27	1050	720	43
MK 5A	475	25	-	135	0.27	1050	720	43
MK 10A	450	50	-	135	0.27	1050	715	43
MK 15A	425	75	-	135	0.27	1050	710	43
SF 5A	475	-	25	135	0.27	1050	725	43
SF 10A	450	-	50	135	0.27	1050	715	43
SF 15A	425	-	75	135	0.27	1050	715	43
Series B (W/CM = 0.30)								
CB	500	-	-	150	0.30	1050	695	19
MK 5B	475	25	-	150	0.30	1050	690	19
MK 10B	450	50	-	150	0.30	1050	685	19
MK 15B	425	75	-	150	0.30	1050	680	19
SF 5B	475	-	25	150	0.30	1050	685	19
SF 10B	450	-	50	150	0.30	1050	680	19
SF 15B	425	-	75	150	0.30	1050	680	19
Series C (W / CM = 0.33)								
CC	500	-	-	165	0.33	1050	700	12
MK 5C	475	25	-	165	0.33	1050	695	12
MK 10C	450	50	-	165	0.33	1050	690	12
MK 15C	425	75	-	165	0.33	1050	685	12
SF 5C	475	-	25	165	0.33	1050	690	12
SF 10C	450	-	50	165	0.33	1050	685	12
SF 15C	425	-	75	165	0.33	1050	680	12

2.4 Testing

The concretes were tested for cube compressive strength (BS 1881: Part 103: 1983), splitting tensile (BS 1881: Part 117: 1983), flexural tensile (BS 1881: Part 118: 1983) and modulus of elasticity (BS 1881: Part 121: 1983) at ages 3, 7, 28, 56 and 90 days in a wet condition. To reduce experimental errors, the specimens for testing were all cast from the same batch of concrete. At least three cubes were tested at each age to compute the average compressive strength. Additional cubes were tested when the deviation of any individual strength value exceeded 3% from the mean value, and the new average was computed based

on three closest strength results.

For splitting strength, flexural strength and static modulus of elasticity in compression, two specimens were tested at each age. Splitting test was performed on cylinders while flexural strength was determined from prisms tested at a third-point loading. For modulus of elasticity, cylindrical specimens were loaded to a maximum of its one-third compressive strength. This value was estimated from the cube strength result, which was converted to equivalent cylinder strength by multiplying a factor of 0.8. Cylinder specimens were ground at the ends to ensure a uniform surface condition. A compressometer, with an effective gauge length of 150 mm and a dial gauge extensometer, with a sensitivity of 0.001 mm were used as a strain measurement apparatus. A digital compression-testing machine, with a maximum capacity of 2000-kN incorporating a 50-kN flexural testing machine was used.

3. RESULTS AND DISCUSSIONS

The experimental data represents concretes with compressive strength values ranging from approximately 45 to 110 MPa at ages 3 to 90 days. The corresponding ranges for other test results are as follows: 3 to 6 MPa (splitting tensile), 5 to 11 MPa (flexural tensile) and 29 to 43 GPa (elastic modulus). The average coefficients of variation for the compression, splitting tensile, flexural tensile and elastic modulus test were approximately 1%, 2.7%, 2.7% and 1.4% respectively. A lower consistency in the tensile strength data is possibly due to the fact that only two specimens were tested per age and also to the inherently high variability associated with these tests.

3.1 Relationship between tensile and compressive strength

As can be seen from Table 1, the various empirical equations that have been formulated to relate tensile strength (f_t) and compressive strength (f_c) are generally expressed in the form of a two-parameter power-function $f_t = k(f_c)^n$ where k and n are coefficients of correlation. The values for n between ½ and ¾ have been suggested. For instance, the most commonly used equations for estimating tensile splitting strength (f_{sp}) and flexural tensile strength (f_r) of normal-weight high-performance concrete, from cylinder compressive strength (f_{cyl}) are those recommended by ACI 363R-92 [2]:

$$f_{sp} = 0.59\sqrt{f_{cyl}} \quad 21MPa < f_{cyl} < 83MPa \tag{1}$$

$$f_r = 0.94\sqrt{f_{cyl}} \quad 21MPa < f_{cyl} < 83MPa \tag{2}$$

These equations were based on a study conducted by Carrasquillo, Nilson and Slate [7] in 1981, on moist-cured specimens tested at the ages of 7, 28 and 95 days.

Data from the present work was used to determine the most suitable empirical equation to relate tensile and cube compressive strength (f_{cu}). The following equations were utilised;

$$f_t = k\sqrt{f_{cu}} + c \tag{3}$$

$$f_t = k\sqrt{f_{cu}} \tag{4}$$

$$f_t = k(f_{cu})^n + c \tag{5}$$

$$f_t = k(f_{cu})^n \tag{6}$$

$$f_t = (f_{cu})^n \tag{7}$$

The correlation coefficients for the above were determined using a non-linear least squares fit method via statistical computer programme, the results of which are summarised in Table 5. Estimates for the best-fit parameters as well as standard deviations and the coefficients of regression (R^2) are given. The curve fitting process was based on results obtained from all mixtures at 3, 7, 28, 56 and 90 day measurements, which covers a total of 105 sets of individual data from approximately 750 specimens.

Table 5: Parameter estimates for various tensile-compressive strength relationships

Equation	**Parameters ***			**R^2**
	k	**n**	**c**	
Splitting tensile strength				
a) $f_{sp} = k\sqrt{f_{cu}} + c$	0.782 *(0.033)*	0.5	-2.027 *(0.3014)*	0.848
b) $f_{sp} = k\sqrt{f_{cu}}$	0.559 *(0.004)*	0.5	-	0.778
c) $f_{sp} = k(f_{cu})^n + c$	0.311 *(0.824)*	0.652 *(0.454)*	-0.4412 *(3.616)*	0.847
d) $f_{sp} = k(f_{cu})^n$	0.222 *(0.032)*	0.709 *(0.032)*	-	0.846
e) $f_{sp} = (f_{cu})^n$	-	0.368 *(0.002)*	-	0.417
Flexural tensile strength				
f) $f_r = k\sqrt{f_{cu}} + c$	1.858 *(0.047)*	0.5	-8.485 *(0.420)*	0.940
g) $f_r = k\sqrt{f_{cu}}$	0.926 *(0.009)*	0.5	-	0.701
h) $f_r = k(f_{cu})^n + c$	0.349 *(0.498)*	0.783 *(0.253)*	-2.673 *(3.357)*	0.943
i) $f_r = k(f_{cu})^n$	0.078 *(0.010)*	1.058 *(0.028)*	-	0.944
j) $f_r = (f_{cu})^n$	-	0.483 *(0.002)*	-	0.685

* The standard deviation of each estimated parameter is shown in parentheses.

From the analyses, it was found that the two-parameter power function in the form of $f_t = k(f_{cu})^n$ gave the best representation for both splitting and flexural strength. Although the $f_t = k(f_{cu})^n + c$ equation also provided good results, however, it cannot be applied over the entire range of strength due to the non-zero intercept c. The low R^2 value for the square root regression function $f_t = k\sqrt{f_{cu}}$ indicates that the model is not a good description of the relationship between the independent and dependent variables. Therefore, the best-fit equations for splitting tensile and flexural tensile based on data from the current study are:

$$f_{sp} = 0.22(f_{cu})^{0.71} \qquad 45MPa < f_{cu} < 110MPa \tag{8}$$

$$f_r = 0.078(f_{cu})^{1.06} \qquad 45MPa < f_{cu} < 110MPa \tag{9}$$

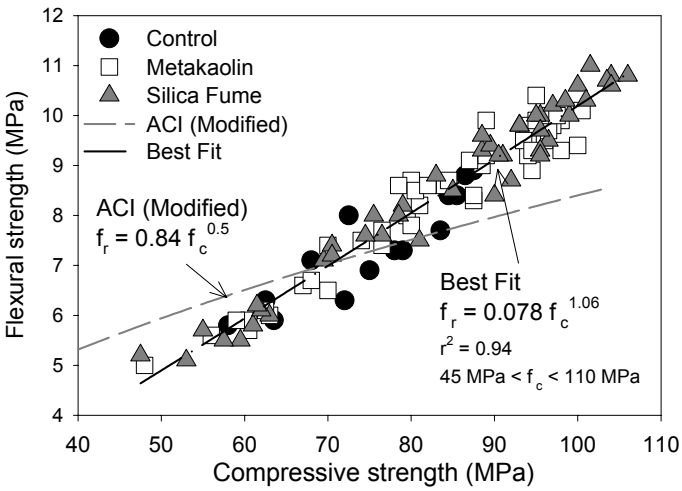

Figure 1. Best-fit curves for splitting and flexural strength

In Figure 1, the best-fit curves for predicting splitting and flexural strength are shown. The ACI formulas, modified to take into account of the difference between cube and cylinder strength are also provided for comparison. Assuming that the cylinder compressive strength f_{cyl} is equivalent to 0.8 of the cube compressive strength f_{cu}, hence the modified ACI equations are:

$$\text{ACI 363 (Modified):} \quad f_{sp} = 0.53\sqrt{f_{cu}} \qquad 26MPa < f_{cu} < 104MPa \tag{10}$$

$$\text{ACI 363 (Modified):} \quad f_r = 0.84\sqrt{f_{cu}} \qquad 26MPa < f_{cu} < 104MPa \tag{11}$$

The result shows that the assumed proportionality of tensile strength to the square root of compressive strength is inaccurate when applied to a wider compressive strength range. In Figure 1, the expressions found in the ACI Code are shown to be low estimates for both splitting and flexural strength, when compressive strength higher than 70 MPa is considered, that is the average strength level normally achieved after 7 days of curing for specimens in this study. At early ages, the ACI formula overestimates tensile strength. This trend was observed regardless of the method used in measuring tensile strength.

It could be argued that the difference between ACI and best-fit equations obtained from this study is due to the size effect of specimens used. The ACI equations suggested by Carrasquillo e*t al*. [7] were obtained from splitting test made on 100ϕ x 200 mm cylinders and flexural tests on 100 x 100 x 350 mm beams. The concretes were made of ASTM Type I cement with no mineral addictives, crushed limestone and gravel as coarse aggregates. Moreover, the validity of cylinder-to-cube compressive strength ratio of 0.8 is also subjected to debate. However, research findings have consistently indicated that the 0.5 power relationship adopted by ACI does not agree particularly well with test results. A summary of various tensile-compressive strength relationships proposed in past studies are shown in Table 6. The data includes concretes made with various cementitious materials, aggregate types, curing regimes and testing conditions; hence is believed to be well represented. Yet, it is surprising to find that, despite all the variability involved, the best-fit power function is usually close to 0.7.

Since most studies did not provide coefficients of regression from their curve-fitting process, it is difficult to determine which relationship is the 'more accurate'. Also, some authors have tended to choose common power functions such as 1/2 or 2/3 rather than to allow the power *n* as a variable in correlating their data. For example, Cetin and Carrasquillo [17] studied the mechanical properties of HPC at constant W/C ratio of 0.28 made with various coarse aggregates (crushed river gravel, trap rock, dolomitic and calcitic limestone) used in varying volumetric contents of 36%, 40% and 44%. They proposed the equation $f_r = 0.83(f_{cyl})^{0.5}$ for flexural strength even though their data clearly shows that the square root function is not the best representation. It overestimates flexural strength at compressive strength levels greater than 50 MPa and underestimates it at higher strength levels. A better fit would have been achieved if the authors used a higher power function.

3.2 Relationship between elastic modulus and compressive strength

Data obtained from the present study was used to determine a suitable empirical relationship between stiffness and cube compressive strength. A method similar to the preceding section was used in the curve fitting process and the computed parameter estimates are summarised in Table 7. With the exception of $E = (f_{cu})^n$, all other equations tested produced good correlation. In fact, the first four equations gave approximately the same coefficient of regression value (R^2) of 0.87. However, equations (a) and (c) were again disregarded since they generate a non-zero intercept c which limits their applicability. By comparing equations (b) and (d), it is interesting to note that both produced very similar results. As such, the former equation with a power factor of 0.5 was chosen as the best representation in

view of its convenience and simplicity.

Table 6: Proposed tensile-compressive strength relationships from various studies

Author(s)	Proposed equation	Notes
Gardner & Poon, [8], 1976	$f_r \ \alpha \ f_{cyl}^{0.8}$ $f_{cyl} < 40$ MPa	Tested concretes made of Type I and Type III cements and cured at 2°C, 13°C and 22°C.
Carino & Lew [9], 1982	$f_{sp} = 0.27(f_{cyl})^{0.71}$ $f_{cyl} < 40$ MPa	Analysed 124 data from various sources. Specimens cured at 2°C, 13°C, 23°C and 32°C. A 19-mm crushed limestone aggregate was used.
Raphael [10], 1985	$f_{sp} = 0.31(f_{cyl})^{0.67}$ $f_r = 0.42(f_{cyl})^{0.67}$ $f_{cyl} < 65$ MPa	Examined 12,000 individual results from four studies conducted between 1928 and 1965, involving a wide variety of concrete of various W/B ratios and aggregate sizes. Specimens of different sizes were tested.
Shah & Ahmad [11], 1985	$f_{sp} = 0.462(f_{cyl})^{0.55}$ $f_r = 0.437(f_{cyl})^{0.67}$ $f_{cyl} < 85$ MPa	Summarised tensile strength data from 6 sources, which includes concretes with strength up to 85 MPa. Different types and sizes of specimens were tested.
Parott [12], 1988	$f_{sp} = 0.226(f_{cyl})^{0.705} + 1$ $f_{cyl} < 120$ MPa	Examined data from Shah & Ahmad [14] and additional data from 7 other references. Concrete strengths up to 120 MPa.
cont'		
Author(s)	Proposed equation	Notes
Oluokun *et al.*[13], 1991	$f_{sp} = 0.584(f_{cyl})^{0.79}$ $f_{cyl} < 62$ MPa	Concretes with W/B ratios 0.33 to 0.76, Type I cement, limestone aggregates and moist cured at 23°C.
Oluokun [14],1991	$f_{sp} = 0.29(f_{cyl})^{0.69}$ $f_{cyl} < 62$ MPa	Reviewed 566 data from ten sources spread over 20 years. Data includes concretes of various W/B ratios, aggregate types, curing regimes and specimen sizes.
Légeron & Paultre [15], 2000	$f_r = 0.50(f_c)^{0.67}$ $f_{cyl} < 130$ MPa	Reviewed 395 flexural strength data points obtained from 22 sources. W/B ratios: 0.21 to 0.76; aggregates: basalt, granite, limestone, quartz; and pozzolans: fly ash, silica fume. Prisms of different sizes were tested.

Zheng *et al.* [16], 2001	$f_{sp} = 0.32(f_{cu})^{0.56}$ $f_{cu} < 70$ MPa	Tested 200 prisms made with concretes of W/B ratio 0.39 to 0.80. Crushed granite and fly ash was used. Binder content between 265 to 460 kg/m^3.

Table 7: Parameter estimates for various stiffness-compressive strength relationships

Equation	Parameters *			R^2
	k	n	c	
a) $E = k\sqrt{f_{cu}} + c$	4.026 *(0.143)*	0.5	1.037 *(1.310)*	0.866
b) $E = k\sqrt{f_{cu}}$	4.138 *(0.013)*	0.5	-	0.865
c) $E = k(f_{cu})^n + c$	1.980 *(4.220)*	0.615 *(0.359)*	7.708 *(16.68)*	0.864
d) $E = k(f_{cu})^n$	4.452 *(0.356)*	0.484 *(0.018)*	-	0.866
e) $E = (f_{cu})^n$	-	0.818 *(0.001)*	-	0.512

*** The standard deviation of each estimated parameter is shown in parentheses.**

In Figure 2, the best-fit curve for elastic modulus is compared to the BS 8110, ACI 318 and ACI 363 equations. The ACI equations were modified by multiplying a factor of $\sqrt{0.8}$ due to the measured strength differences between a cube and cylinder specimen.

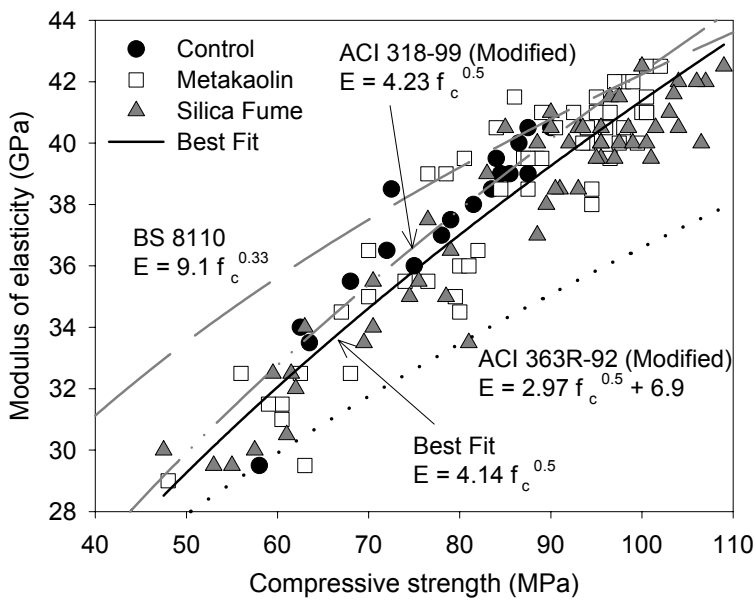

Figure 2. Best-fit curve for static modulus of elasticity

$$\text{ACI 318 (Modified.): } E = 4.23\sqrt{f_{cu}} \quad f_{cu} < 50MPa \tag{12}$$

$$\text{ACI 363 (Modified): } E = 2.97\sqrt{f_{cu}} + 6.9 \quad 26MPa < f_{cu} < 104MPa \tag{13}$$

It can be seen that the modified ACI 318 equation performed quite well in predicting the experimental values, albeit slightly higher than average values; this despite the fact that the equation was originally derived from normal-strength concrete. On the other hand, the ACI 363 equation, which has been suggested for strengths up to 83 MPa seems inappropriate, drastically underestimating stiffness at strengths higher than 50 MPa. This was also reported in studies done by Cetin and Carrasquillo [17]; and Mesbah *et al.* [18]. The BS 8110 equation also failed to provide good prediction, overestimating elastic modulus at low compressive strengths, even though it was designed for strengths up to 60 MPa.

Various attempts have been made to relate stiffness with other properties such as concrete density and factors related to the type of coarse aggregates used. In 1960, Pauw [19] tested concretes with cylinder strength of between 25 to 40 MPa and recommended the equation $E = 4.3 \times 10^{-5}\ W_c^{1.5}\sqrt{f_{cyl}}$ where W_c is the air-dry unit weight of the concrete at the time of test (1500-2500 kg/m^3). Assuming that the air-dry unit weight of the concrete used in this study was 2300 kg/m^3 and correcting for cylinder strength, Pauw's equation can be simplified to $E = 4.24\sqrt{f_c}$, which correlates remarkably well with present findings. Iravani [20] tested the stiffness of HPC (with and without SF) at 56 days with cylinder compressive strength in the range of 55 to 125 MPa and found the best-fit relationship was $E = 3.375\sqrt{f_{cyl}}$. Subsequently, by combining his results with selected data from the literature, proposed a modified equation in the form $E = 4.7C_{ca}\sqrt{f_{cyl}}$ where C_{ca} is an empirical coefficient depending on the type of coarse aggregate. By using a C_{ca} value of 0.82 for granite (recommended by Iravani) and again correcting for cylinder strength, the equation becomes $E = 3.45\sqrt{f_c}$, which is conservative with respect to the present findings.

3.3 Discussion

The study shows that unlike in the tensile-compressive strength relationship, a square-root function can be used with good accuracy for estimation of stiffness. For splitting tensile strength and flexural strength however, a higher power function of 0.71 and 1.06 is required. Traditionally, the square root function was chosen out of convenience, but with the wide-availability of hand-held calculators, there is no reason why a higher power function cannot be adopted for tensile strength prediction if it provides a much accurate result. Another interesting point to note is that inclusion of metakaolin or silica fume did not have any significant effects on the trend of both tensile-compressive strength and stiffness-compressive strength relationships. At a particular strength level, blended mixtures did not appear to consistently achieve higher (or lower) tensile strength and stiffness than the control. This means that both properties can be directly related to compressive strength,

without needing an additional factor to account for any effects of the mineral admixtures.

In comparing the various equations available from literature, it should be cautioned that the data includes studies with and without supplementary cementitious materials; different mixture proportions, coarse aggregate types, curing regimes, age of testing and specimen sizes. The comparison given is also subjected to the accuracy of the cylinder to cube strength conversion factor. According to BS 1881: Pt. 120: 1983, the strength of cylindrical specimen is approximately equal to 0.8 of the strength of a cube, but in reality there is no simple relationship between the strengths obtained from specimens of the two shapes. It has been reported that the ratio increases strongly with an increase in compressive strength and is close to 1 at strengths of more than 100 MPa [21]. The CEB-FIP Design Code [3] gives a table of equivalence of strengths for the two types of compression specimens, but above 50 MPa, the cylinder-cube strength ratio increases progressively from 0.8 and reaches 0.89 when the cylinder strength is at 80 MPa.

In selecting a best-fit mathematical function for a widely distributed experimental data as in the case of concrete materials, one has to decide whether to follow an average approach or a 'safe value' such that a minimum number of tests will fall below the chosen equation. In view of the differences in materials and testing conditions in the laboratory compared to the construction site, it may be justifiable to have a predictive equation that produces a lower value for safety reasons. However, in actual practice, the concrete designer will consider the test distribution values and then assign a mixture with average strength higher than the required design strength by a factor depending on expected quality control. Hence, to use a 'safe' predictive equation would compound safety factors and result in material wastage.

It is not the intention of this paper to derive a definitive set of equations relating compressive strength with tensile strength and elastic modulus for all types of concretes. This objective would be too ambitious for a limited study like the present one. Indeed, there can be no unique relationship for all HPC, particularly in the case of elastic modulus since it is significantly affected by the type and volumetric proportion of the coarse aggregate used. HPC mixtures with low water-cement ratios result in much improved cement paste and transition zone, to a stage where the coarse aggregates now becomes a limiting factor. Hence, empirical relationships should be used with caution and be viewed as only valid in general terms; that is only if common mixture designs and materials are selected.

4. CONCLUSIONS

Despite its importance, tensile strength (and elastic modulus) is not usually measured in the site for compliance purposes; but is often estimated from the measured compressive strength based on empirical relationships proposed by various codes of practice. The purpose of this experimental programme was to re-examine some of these relationships by using new data obtained from mixtures containing a laboratory produced metakaolin and commercial silica fume. The concretes investigated have strengths up to 110 MPa at 90 days. It was found that at a particular strength level, blended mixtures did not appear to consistently achieve higher (or lower) tensile strength and stiffness than the control, hence both properties can be directly related to compressive strength, without needing any additional factor to account for

the mineral admixtures. It is thus realised that the square-root function adopted by most codes of practice for tensile strength is inadequate when applied to high-performance concrete. The current ACI equations overestimate tensile strength at early ages and as such, may be unsafe when applied to young concrete. Based on the results of this study, a higher power function of 0.71 and 1.06 was found to be more suitable for splitting and flexural strength. For elastic modulus, the square-root function proposed by ACI 318 performed far better than the ACI 363 equation, though the later was designed for high-strength concrete. Although strength relationships can be used effectively for ordinary concretes of the past, with the emergence of new, exotic and designer concretes for the niche market, these predictive relationships should be used with caution. All this imply that there is a need for current standards to develop a new set of relationships for a wide range of concretes and to stress on its range of application.

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NOTATIONS

f_c compressive strength (MPa)
f_t tensile strength (MPa)
f_{cu} cube compressive strength (MPa)
f_{cyl} cylinder compressive strength (MPa)
f_{sp} tensile splitting strength (MPa)
f_r flexural tensile strength (MPa)
E modulus of elasticity (GPa)